Summer 2017

# Radiation Reaction: Or how I learned to stop worrying and love E&M

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## Introduction

Classical E&M

- a consistent relativistic field theory
- described by Maxwell's equations and the Lorentz force equation

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

A consequence of classical E&M:

accelerating charges produce light.

This can be seen as synchrotron radiation.

We examine the Abraham-Lorentz-Dirac equation (ALD) without modification, describing the trajectory of a charged particle undergoing radiation, for physically relevant information.

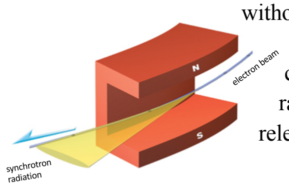



Fig 1. A depiction of synchrotron radiation. [4]

## Abraham-Lorentz-Dirac Equation

The energy loss due to radiation can be modeled as work done by an effective force.

This conservation of energy argument leads to the ALD.

$$ma^{\alpha} = qF^{\alpha\beta}u_{\beta} + \frac{q^2}{6\pi\epsilon_0 c^3}\left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)\dot{a}^{\beta}$$

$$m\mathbf{a} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \frac{q^2}{6\pi\epsilon_0 c^3}\dot{\mathbf{a}}$$

- This equation requires initial position, velocity and acceleration.
- It exhibits runaway solutions where acceleration increases indefinitely.
- Specifying that the final acceleration goes to zero introduces preacceleration.
- The Landau-Lifschitz approximation has neither runaway solutions nor preacceleration.
- Modeling the particles with a finite size results in solutions with neither runaways or preacceleration.

## Approach 1: No Restrictions

Without modification, solutions to the ALD exhibit displeasing features beyond runaway solutions, specifically:

- Charges of opposite sign repel
- Charges of like sign attract

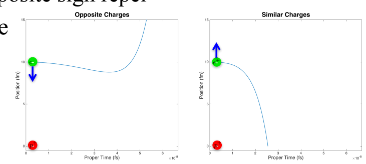



Fig 5. Graphs showing solutions for ALD with opposite charges repelling and like charges attracting. Green particles follow the path in blue as time progresses. Red particles are stationary. Arrows indicate direction of coulomb force on green particle due to red particle.

## Approach 2: $a(\tau \to \infty) = 0$

Requiring the acceleration to vanish at long times introduces preacceleration.

It also is difficult to use with spatially dependent forces.

$$ma(\tau) = b\int_{t}^{\infty} e^{-b(\tau'-\tau)} f(\tau')d\tau'$$

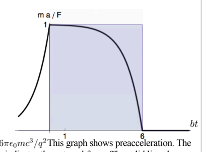



Fig 3. $b = 6\pi\epsilon_0 mc^3/q^2$ This graph shows preacceleration. The shaded area indicates the external force. The solid line shows the acceleration of a particle under ALD. Adapted from [2].

## Approach 3: Landau-Lifshitz

This is an approximation of ALD which assumes that the energy radiated is small.

$$\dot{\mathbf{a}} = \frac{q}{m}\frac{d}{dt}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

$$m\mathbf{a} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \frac{q^3}{6\pi\epsilon_0 mc^3}\frac{d}{dt}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Why should we trust an approximation but not the equation that it approximates?

## Approach 4: Particles with size

By giving the particles size we can introduce a time delay between when either side "feels" the force.

If an electron has radius $R \geq 1.879\mathrm{fm}$ (*larger* than a real electron), there are no pathologies.

$$R \geq \frac{q^2}{6\pi\epsilon_0 mc^2}$$

$$\mathbf{F} = \mathbf{F}_{\text{ext}} + \frac{q^2}{12\pi\epsilon_0 R^2 c}\left[\mathbf{v}\left(t - \frac{2R}{c}\right) - \mathbf{v}\left(t\right)\right]$$

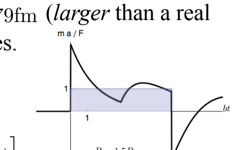



Fig 4. The shaded area indicates the external force. The solid line shows the acceleration of a particle with a finite radius. Adapted from [2].

## Nondimensionalization

We performed calculations for a particle in an attractive Coulomb field in MATLAB.

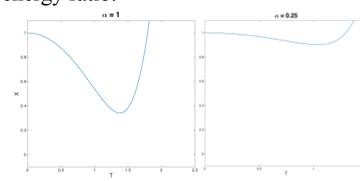
In order to find solutions we nondimensionalize the equation and the ALD becomes,

$$X'' = -\frac{\alpha}{X^2} + \frac{2}{3}\alpha X''' \qquad \alpha = \frac{q^2/4\pi\epsilon_0 x_0}{mc^2}.$$

This left us with the parameter α, a characteristic electrostatic/rest energy ratio.

We set the initial acceleration by the Coulomb force, so that

$$X''(0) = \frac{-\alpha}{X(0)^2}.$$



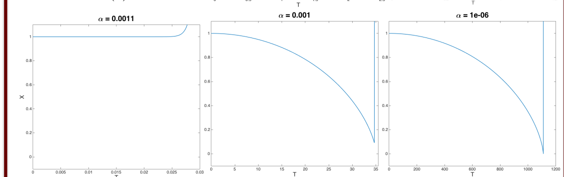




Fig 5. Graphs of nondimensionalized numerical solutions for decreasing values of α.

## Minimum approach

For electron/proton attraction $x_0 = \frac{2.8179\mathrm{fm}}{\alpha}$.

- Above the red line as initial separation increases so does the minimum approach.
- Below, the minimum approach is cut off before runaway behavior takes over
- Is this numerical cutoff physically meaningful?

| $\alpha$ | $x_0$ (fm) | $x_{min}$ (fm) |
|---|---|---|
| 1 | 2.81794 | 0.9673 |
| 0.5 | 5.6359 | 3.9342 |
| 0.25 | 11.2718 | 10.1733 |
| 0.1 | 28.1794 | 27.7541 |
| 0.01 | 281.794 | 281.779 |
| 0.005 | 563.588 | 563.583 |
| 0.0011 | 2561.76 | 2561.76 |
| 0.001 | 2817.94 | 260.447 |
| 0.0001 | 28179.4 | 245.171 |
| 0.000001 | 2817940 | 250.911 |

## Acknowledgements

Special thanks to David Latimer, the University of Puget Sound, the John M. Gilbertson Foundation, and Joel Franklin for making this research possible.

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[2] D. J. Griffiths *et al.,* American Journal of Physics **78**, 391 (2010); doi: http://dx.doi.org/10.1119/1.3269900
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